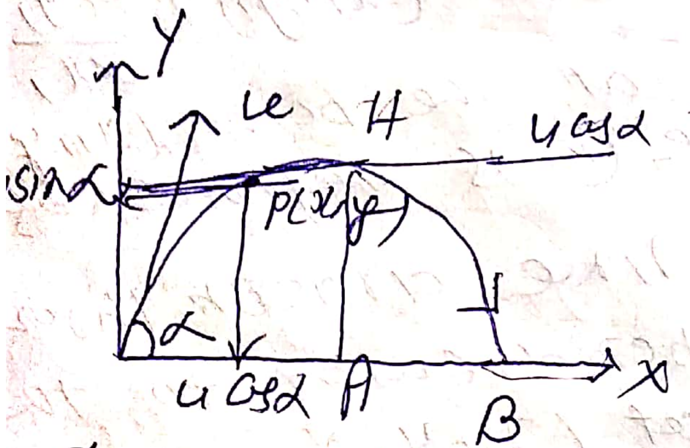
BSC (Math) – part II paper IV

Topic → Projectile motion in non-resisted medium

Theorem Let a particle of mass m is freely projected under the action of gravity in non-resisting medium with a velocity u in a direction making angle α with horizontal. Find (i) motion (ii) the path described (iii) time of flight (iv) maximum height (v) horizontal range (vi) maximum horizontal range.

Ans –



Suppose an object be projected from the point O with velocity u making angle α with the horizontal direction OX. Initially the horizontal and vertical component are u cos α and u sin α respectively.

Let the position of the particle at any time t be P(x,y)

The Components of acceleration of particle at time t are $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$ vertical upward

Hence the equation of motion are $m\frac{d^2x}{dt^2} = 0$ — (1)

and $m\frac{d^2y}{dt^2} = -mg$ — (2)

as the weight mg of the particle is acting vertically downwards.

integ (1) and (2) w.r.to t

we get $\frac{dx}{dt} = A$ and $\frac{dy}{dt} = -gt+C$

integ a second time — (3)

$x = At+B$ and $y = -g\frac{t^2}{2}+Ct+D$

Initially $t=0, x=0, y=0$ — (4)

$\frac{dx}{dt} = u\cos\alpha$, $\frac{dv}{dt} = u\sin\alpha$

Hence from (3) and (4) we have

Initially $u\cos\alpha = A$, $u\sin\alpha = C$, $0=B$ and $0=D$

∴ (4) gives $x = u\cos\alpha . t$ ie $t = \frac{x}{u\cos\alpha}$

and $y = u\sin\alpha t - \frac{1}{2}gt^2$ — (5)

ie $y = u\sin\alpha \frac{x}{u\cos\alpha} - \frac{1}{2}g.\frac{x^2}{u^2\cos^2\alpha}$

$= u\tan\alpha - \frac{g}{2}\frac{x^2}{u^2\cos^2\alpha}$ — (6)

Hence the path of ~~projectile~~ projectile projected horizontally from ground a parabolic path

Time of flight :- It is the total time for which the object is flight while going from O to B. It is denoted by T.

By (5) we put $y = 0$

$$0 = t(u\sin\alpha - \frac{1}{2}gt)$$

$$\Rightarrow u\sin\alpha = \frac{1}{2}gt \quad (\because t \neq 0)$$

$$\Rightarrow t = \frac{2u\sin\alpha}{g}$$

$$\therefore T = t = \frac{2u\sin\alpha}{g}$$

Maximum height

It is the maximum vertical height attained by the object above the point of projection during its flight. It is denoted by h

So we put $y = h$ and $t = \frac{u\sin\alpha}{g}$ in (5)

we get $h = \frac{u\sin\alpha}{g} \cdot u\sin\alpha - \frac{1}{2}\frac{u^2 g \sin^2\alpha}{g^2}$

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$h = \frac{u^2 \sin^2 \alpha}{g} - \frac{u^2 \sin^2 \alpha}{2g}$

$\therefore$ Max. height $= h = \frac{u^2 \sin^2 \alpha}{2g}$

Horizontal Range :- It is the horizontal distance by the object between its point of projection and the point of hitting the ground. It is denoted by R

$\therefore R = u\cos\alpha \times$ Time of flight

$= u\cos\alpha \times T = u\cos\alpha \cdot \frac{2u\sin\alpha}{g}$

$\therefore R = \frac{u^2 \sin 2\alpha}{g}$

Maximum Horizontal Range

From maximum horizontal Range,

$\sin 2\alpha = 1 = \sin 90^\circ$

$\therefore \alpha = 45^\circ$

$\therefore$ Max. horizontal Range

$R_m = \frac{u^2}{g} \sin 2 \times 45^\circ$

$= \frac{u^2}{g}$

$\therefore R_m = \frac{u^2}{g}$